Solutions
ECE 320
HW #11
Due Monday, April 25th at 5pm

1. (10 pts) In a buck-boost converter, consider all components to be ideal. Let V_{in} be 8-40V, V_o=15V (regulated), f_s=20kHz, and C=470µF. Calculate L_{min} that will keep the converter operating in continuous-conduction mode if $P_o \geq 2W$.

$V_{inmin} := 8V$ $\quad V_{inmax} := 40V$ $\quad V_o := 15V$ $\quad f_s := 20kHz$ $\quad C_1 := 470\mu F$

$P_{omin} := 2W$ $\quad P_{inmin} := 2W$ $\quad T_s := \frac{1}{f_s} = 50 \cdot \mu s$

$I_o := \frac{P_{omin}}{V_o}$ $\quad I_o = 0.133\,A$

Use the equation for Duty Cycle of a buck-boost converter to find the range of D. Note that the minimum duty cycle occurs when the input voltage is a maximum.

$D_{min} := \frac{V_o}{V_o + V_{inmax}}$ $\quad D_{min} = 0.273$ $\quad D_{max} := \frac{V_o}{V_o + V_{inmin}}$ $\quad D_{max} = 0.652$

$I_{inmin} := \frac{P_{inmin}}{V_{inmax}}$ $\quad I_{inmin} = 0.05\,A$

Since the average value of the inductor current is the sum of the output current (constant) and the input current, the inductor current will be a minimum when the average input current is a minimum.

To get the minimum average current through the inductor, that happens when I_L is at a minimum

$I_{Lmin} := I_o + I_{inmin}$ $\quad I_{Lmin} = 0.183\,A$

$\Delta i_L := 2 \cdot I_{Lmin}$ $\quad \Delta i_L = 0.367\,A$

$L_{min} := \frac{V_{inmax}}{\Delta i_L} \cdot D_{min} \cdot T_s$ $\quad L_{min} = 1.488 \cdot mH$

Use this inductor value to check for continuous conduction mode.

$i_{Lmin} := I_{Lmin} - \frac{1}{2} \cdot \frac{V_{inmax}}{L_{min}} \cdot D_{min} \cdot T_s$ $\quad i_{Lmin} = 0\,A$ $\quad$ This is the boundary condition.

To check the other value of inductance that you could calculate:

$$I_{Lmax} := I_o + \frac{P_{inmin}}{V_{inmin}} \qquad I_{Lmax} = 0.383\,A$$

$$\Delta i_{Lmax} := 2 \cdot I_{Lmax} \qquad \Delta i_{Lmax} = 0.767\,A$$

$$L_{other} := \frac{V_{inmin}}{\Delta i_{Lmax}} \cdot D_{max} \cdot T_s \qquad L_{other} = 0.34 \cdot mH$$

Yes, this is smaller, but will it work over the entire range of duty cycles?

$$i_{Lcheck} := I_{Lmax} - \frac{1}{2} \cdot \frac{V_{inmin}}{L_{other}} \cdot D_{max} \cdot T_s \qquad i_{Lcheck} = 0\,A$$

so this works as expected, but then when you use L_{other} with the minimum duty cycle condition:

$$i_{Lcheck2} := I_{Lmin} - \frac{1}{2} \cdot \frac{V_{inmax}}{L_{other}} \cdot D_{min} \cdot T_s \qquad i_{Lcheck2} = -0.618\,A$$

Puts the converter into DCM at minimum duty cycle.

$$i_{Lmin} := I_{Lmax} - \frac{1}{2} \cdot \frac{V_{inmin}}{L_{min}} \cdot D_{max} \cdot T_s \qquad i_{Lmin} = 0.296\,A$$

This is the boundary condition.

2. (12 pts Extra Credit) In a buck-boost converter, V_{in}=12V, V_o=15V, I_o=250mA, L=150μH, C=470μF, and f_s=20kHz.

$V_{in} := 12V$ $V_o = 15\,V$ $I_{o2} := 250mA$ $L_2 := 150\mu H$ $C_2 := 470\mu F$ $f_s = 20\cdot kHz$

$T_s = 50\cdot\mu s$

(a) Calculate ΔV_o (peak-peak)

$$D_2 := \frac{V_o}{V_o + V_{in}} \qquad D_2 = 0.556$$

First need to find I_{OB} to see if we're in continuous conduction mode.

$$I_{oB} := \frac{T_s \cdot V_o}{2 \cdot L_2} \cdot \left(1 - D_2\right)^2 \qquad I_{oB} = 493.8\cdot mA$$

$I_{o2} < I_{oB}$ so the converter is in discontinuous condution mode

Now that we know the converter is in discontinuous conduction mode, can solve for the actual duty cycle using the equation from Lecture #39

When the duty cycle is at a minimum, Io will be a max

$$I_{oBmax} := \frac{T_s \cdot V_o}{2 \cdot L_2} \qquad I_{oBmax} = 2.5\,A$$

$$D_{dcm} := \frac{V_o}{V_{in}} \cdot \sqrt{\frac{I_{o2}}{I_{oBmax}}} \qquad D_{dcm} = 0.395$$

To find the ΔV_o, need to look at the current through the diode. There will only be current through the diode when the switch is open (t_{off}). This will be the same as the value of the current through the inductor at that time. When the switch changes from ton to toff, the current throught inductor will be at it's peak at that time.

$$i_{Lpeak} = \Delta i_L = i_{Dmax} \qquad i_{Dmax} := \frac{V_{in}}{L_2} \cdot D_{dcm} \cdot T_s \qquad i_{Dmax} = 1.581\,A$$

The current through the inductor will decay at a slope of $-V_o/L$

$$slp_{neg} := \frac{-V_o}{L_2} \qquad slp_{neg} = -100\cdot\frac{kA}{s}$$

Next, plot this characteristic along with the average output current.

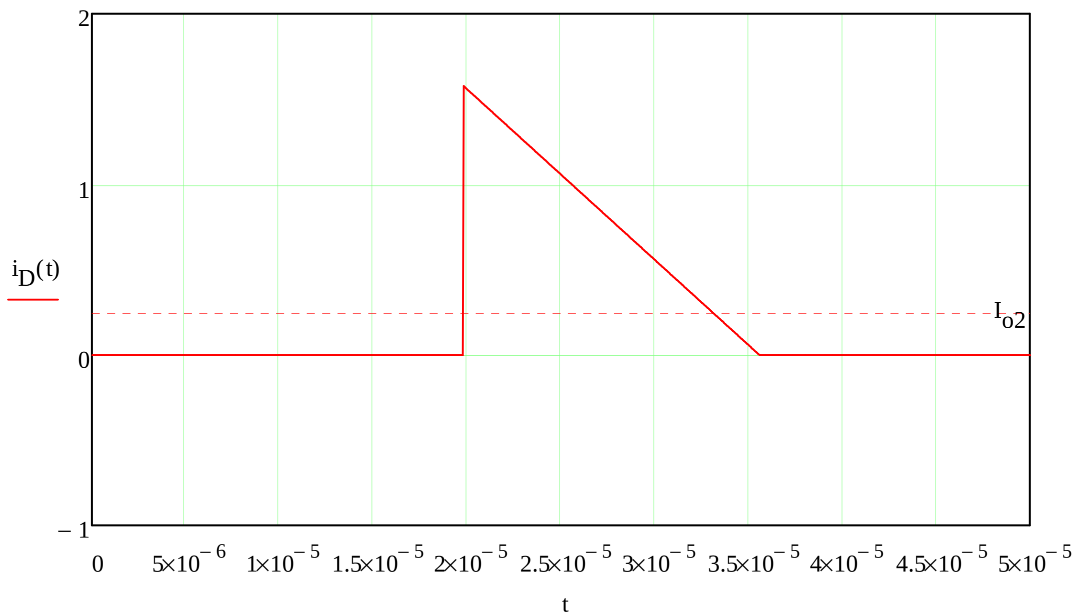
Find the equation of the line of the diode current while the switch is off

$y = mx + b$

$i_{Dmax} = slp_{neg} \cdot D_{dcm} \cdot T_s + b$ $b := i_{Dmax} - slp_{neg} \cdot D_{dcm} \cdot T_s$ $b = 3.558\,A$

find out when i_D=0 $t_0 := \frac{0 - b}{slp_{neg}}$ $t_0 = 35.576 \cdot \mu s$

$$i_D(t) := \begin{cases} 0 & \text{if } 0 < D_{dcm} \cdot T_s \\ (slp_{neg} \cdot t + b) & \text{if } D_{dcm} \cdot T_s \le t < (t_0) \\ 0 & \text{otherwise} \end{cases}$$



Find out at what time the diode current is equal to the output current

$t_2 := \frac{I_{o2} - b}{slp_{neg}}$ $t_2 = 33.076 \cdot \mu s$ $\Delta t := t_0 - t_2 = 2.5 \cdot \mu s$

$\Delta Q_2 := I_{o2} \cdot D_{dcm} \cdot T_s + (T_s - t_0) \cdot I_{o2} + \frac{1}{2} \cdot I_{o2} \cdot \Delta t$ $\Delta Q_2 = 8.86 \times 10^{-6}\,C$

As a check: $\Delta Q_{check} := \frac{1}{2} \cdot (i_{Dmax} - I_{o2}) \cdot (t_2 - D_{dcm} \cdot T_s)$ $\Delta Q_{check} = 8.86 \times 10^{-6}\,C$

From here, we can solve for the ΔV_o

$$\Delta V_o := \frac{\Delta Q_2}{C_2} \qquad \Delta V_o = 18.85 \cdot mV$$

(b) Calculate the rms value of the ripple current through the diode.

$$i_{Drms} := \sqrt{\frac{1}{T_s} \cdot \int_0^{T_s} i_D(t)^2 \, dt} \qquad i_{Drms} = 0.513\,A$$

$$i_{D\_ripple\_rms} := \sqrt{i_{Drms}^2 - I_{o2}^2} \qquad i_{D\_ripple\_rms} = 0.448\,A$$

(c) Derive the expression for ΔV_o (peak-peak) in a discontinuous-conduction mode in terms of the circuit parameters.

Just take one of your equations for ΔQ and simplify it to where it only uses circuit parameter.

$$\Delta V_{o\_expression} := \frac{I_{o2} \cdot \left[T_s \cdot \left(1 - \sqrt{\frac{I_{o2} \cdot 2 \cdot L_2}{T_s \cdot V_o}} \right) + \frac{I_{o2} \cdot L_2}{2 \cdot V_o} \right]}{C_2}$$

As a check:

$$\Delta V_{o\_expression} = 18.85\,mV$$

Same as above, so expression must be right.

(d) If I_o is now equal to $0.5 * I_{oB}$, calculate ΔV_o (peak-peak)

$$I_{onew} := \frac{1}{2} \cdot I_{oB} \qquad I_{onew} = 246.914\,mA$$

$$\Delta V_{o\_new} := \frac{I_{onew} \cdot \left[T_s \cdot \left(1 - \sqrt{\frac{I_{onew} \cdot 2 \cdot L_2}{T_s \cdot V_o}} \right) + \frac{I_{onew} \cdot L_2}{2 \cdot V_o} \right]}{C_2}$$

$$\Delta V_{o\_new} = 18.66\,mV$$